

Prove the derivative of  $\ln x$  using implicit differentiation.

SCORE: \_\_\_\_\_ / 3 PTS

$$y = \ln x$$

$$e^y = x \quad \textcircled{1}$$

$$e^y \frac{dy}{dx} = 1 \quad \textcircled{1}$$

$$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$$

$\textcircled{1}_2$

$\textcircled{2}_2$

Find  $\frac{dy}{dx}$  if  $\ln(2x^2y^3 - y^2) = 1 - y^3$

3 SOLUTIONS GIVEN -  
GRADE AGAINST ONLY 1 SOLUTION

SCORE: \_\_\_\_ / 6 PTS

SOLUTION 1: (NO REWRITE OF EQUATION)

$$\textcircled{1} \quad \frac{1}{2x^2y^3 - y^2} \left( 4xy^3 + 6x^2y^2 \frac{dy}{dx} - 2y \frac{dy}{dx} \right) = -3y^2 \frac{dy}{dx}$$

$$4xy^3 + 6x^2y^2 \frac{dy}{dx} - 2y \frac{dy}{dx} = (3y^4 - 6x^2y^5) \frac{dy}{dx} \textcircled{1}$$

$$4xy^3 = (2y + 3y^4 - 6x^2y^2 - 6x^2y^5) \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{4xy^3}{2y + 3y^4 - 6x^2y^2 - 6x^2y^5} \textcircled{1}$$

$$= \frac{4xy^2}{2 + 3y^3 - 6x^2y - 6x^2y^4} \textcircled{\frac{1}{2}}$$

SOLUTION 2: (USE LOG RULES)

$$\ln((2x^2y-1)y^2) = 1-y^3$$

$$\textcircled{1} \quad \underline{\ln(2x^2y-1) + 2\ln y = 1-y^3}$$

$$\textcircled{1} \quad \boxed{\frac{1}{2x^2y-1}} \left( 4xy + 2x^2 \frac{dy}{dx} \right) + \frac{2}{y} \frac{dy}{dx} = -3y^2 \frac{dy}{dx}$$

$$y(4xy + 2x^2 \frac{dy}{dx}) + 2(2x^2y-1) \frac{dy}{dx} = -3y^3(2x^2y-1) \frac{dy}{dx}$$

$$\boxed{4xy^2 + (2x^2y + 4x^2y - 2) \frac{dy}{dx} = (-6x^2y^4 + 3y^3) \frac{dy}{dx}} \quad \textcircled{1}$$

$$(6x^2y^4 - 3y^3 + 6x^2y - 2) \frac{dy}{dx} = -4xy^2$$

$$\frac{dy}{dx} = \boxed{\frac{-4xy^2}{6x^2y^4 - 3y^3 + 6x^2y - 2}} \quad \text{or} \quad \boxed{\frac{4xy^2}{2 - 6x^2y + 3y^3 - 6x^2y^4}}$$

\textcircled{1} FOR EITHER ONE

### SOLUTION 3: (EXPONENTIATE)

$$\textcircled{1} \boxed{2x^2y^3 - y^2 = e^{1-y^3}}$$

$$\textcircled{2} \boxed{4xy^3 + \boxed{6x^2y^2 \frac{dy}{dx}} - 2y \frac{dy}{dx} = e^{1-y^3} \cdot (-3y^2) \frac{dy}{dx}, \textcircled{1}}$$

$$4xy^3 = (2y - 6x^2y^2 - 3y^2e^{1-y^3}) \frac{dy}{dx}$$

$$\frac{dy}{dx} = \boxed{\frac{4xy^3}{2y - 6x^2y^2 - 3y^2e^{1-y^3}}} \textcircled{2}$$

Find the following derivatives. Simplify all answers appropriately.

① POINT EACH ITEM

SCORE: \_\_\_ / 21 PTS

UNLESS OTHERWISE NOTED

[a]  $\frac{d}{dt} t^{\cot^2 t}$

$$y = t^{\cot^2 t}$$

$$\ln y = \cot^2 t \ln t$$

$$\frac{1}{y} \frac{dy}{dt} = 2 \cot t (-\csc^2 t) \ln t \quad (2)$$

$$+ (\cot^2 t) \frac{1}{t}$$

$$\frac{dy}{dt} = y \cot t \left( \frac{\cot t}{t} - 2 \csc^2 t \ln t \right)$$

$$= t^{\cot^2 t} \cot t \left( \frac{\cot t}{t} - 2 \csc^2 t \ln t \right),$$

$$= t^{\cot^2 t - 1} \cot t (\cot t - 2 t \csc^2 t \ln t)$$

[b]  $\frac{d}{dx} \ln \frac{\sqrt{\cos x}}{(1+\tan x)^5}$

$$= \frac{1}{\sqrt{\cos x}} \left( \frac{1}{4} \ln \cos x - 5 \ln (1+\tan x) \right)$$

$$= \frac{1}{4} \frac{1}{\cos x} \cdot -\sin x - 5 \frac{1}{1+\tan x} \sec^2 x$$

$$= -\frac{1}{4} \tan x - \frac{5 \sec^2 x}{1+\tan x} \quad (2)$$

[c]  $\frac{d}{d\theta} \cos^{-1} \frac{\theta}{1-\theta}$

$$= - \left| \frac{1}{\sqrt{1 - \left( \frac{\theta}{1-\theta} \right)^2}} \right| \frac{1-\theta - \theta(-1)}{(1-\theta)^2}$$

$$= - \frac{1}{\sqrt{\frac{1-2\theta+\theta^2-\theta^2}{(1-\theta)^2}}} \left| \frac{1}{(1-\theta)^2} \right| \quad (2)$$

$$= - \frac{1}{\sqrt{1-2\theta}} \cdot \frac{1}{(1-\theta)^2}$$

$$= - \left| \frac{1}{|1-\theta| \sqrt{1-2\theta}} \right|$$

[d]  $\frac{d}{dy} \tan^{-1} (y + \sec^2 y)$

$$= \frac{1}{1+(y+\sec^2 y)^2} (1+2\sec y \cdot \sec y \tan y)$$

$$= \frac{1+2\sec^2 y \tan y}{1+(y+\sec^2 y)^2} \quad (2)$$